

MAS 117

Introduction to Algebra

Assignment 10

For handing in on 28 March 2007

*Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the **RED** post-box on the **BASEMENT** of the Maths building before 16:00 on Wednesday.*

- 1** (25 marks) Write the following element of S_{10} in cycle notation, and calculate its order:

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 4 & 10 & 9 & 7 & 3 & 5 & 1 & 8 & 6 & 2 \end{pmatrix}.$$

- 2** (25 marks) Let g be the permutation in the preceding question, and let

$$h = (1, 5, 9, 7, 4)(2, 3, 6)(8, 10).$$

Calculate h^{-1} , gh , $h^{-1}gh$.

- 3** (25 marks) Let G be the symmetric group S_3 .

- (a) What is the order of G ?
- (b) What are the orders of elements of G ?
- (c) Show that G contains just one subgroup of order 3 and three subgroups of order 2.

- 4** (25 marks) Show that a permutation which has odd order must be an even permutation.

Is the converse true?

5 Prize question A cash prize is offered for the best solution received to this question. Do not hand this in with the rest of your coursework; give your solutions to me.

For which of the following sets and operations is it true that $f \circ g = e$ implies $g \circ f = e$, where e is the identity?

- (a) X is the set of all functions from $\{1, \dots, n\}$ to itself, $\circ$ is composition.
- (b) X is the set of all functions from an arbitrary set X to itself, $\circ$ is composition.
- (c) $X = M_{n \times n}(\mathbb{R})$ (the set of $n \times n$ real matrices), $\circ$ is matrix multiplication.